ALGEBRA II | Homework 1130 | Name:
DR. PAUL L. BAILEY | Tuesday, November 30, 2021

Due Wednesday, November 30, 2021. Do not copy. Do not write anything you do not understand.

Definition 1. Let A and B be sets. A *function* from A to B is an assignment of every element in A to a unique element in B. We say that f *maps* A *into* B.

Let f be a function from A to B. If $a \in A$, the element of B to which a is assigned by f is denoted $f(a)$. Functions satisfy this "defining property":

$$\text{for every } a \in A \text{ there exists a unique } b \in B \text{ such that } f(a) = b.$$

If f is a function from A to B, this fact is denoted

$$f : A \to B.$$

- The *domain* of f is A.
- The *codomain* of f is B.
- The *range* of f is $f(A) = \{b \in B \mid b = f(a) \text{ for some } a \in A\}$.

We say that f maps A *onto* B if $f(A) = B$.

Problem 1. For each of the following situations, determine if the assignment is a function from A to B. Explain your reasoning. If "it depends", say what it depends on. If it is a function, state whether it is "onto".

(a) A is the set of fish in a pet store, B is the set of fish tanks in the store, assign a fish to its tank.

(b) A is the set cars in the Paragon parking lot, B is the set of students in the school, assign a car to the student that the car delivered.

(c) A is the set positive integers, B is the set of positive integers, a is assigned to b if $a = 2b + 1$.

(d) A is the set of stars in the universe, B is the set of galaxies in the universe, assign a star to the galaxy it is in.

(e) A is the set of days in the last five years, B is the set nonnegative integers, R is a set of people, each day is assigned to the number of lies told by someone in R on that day.

(f) A is the set of desks in the classroom, B is the set of students in the classroom, assign each desk to the student who sits there.

Definition 2. A *rational function* is a function of the form $f(x) = \dfrac{p(x)}{q(x)}$, where p and q are polynomials.

A *linear fractional transformation* is a function of the form $f(x) = \dfrac{ax+b}{cx+d}$, where $a, b, c, d \in \mathbb{R}$.

Problem 2. Find the domain of the function f. Express your answer using correct set notation.

(a) $f(x) = x^3 - 7x + 2$

(b) $f(x) = \dfrac{1}{x-1}$

(c) $f(x) = \dfrac{x-2}{x-1}$

(d) $f(x) = \dfrac{x-1}{x^2-4}$

(e) $f(x) = \sqrt{x}$

Problem 3. Find the domain and range of the linear fractional transformation f.

(a) $f(x) = \dfrac{1}{x-1}$

(b) $f(x) = \dfrac{3}{x+5} + 7$

(c) $f(x) = \dfrac{x-2}{x-1}$